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Radiative-transter effects on thermal-convective instability in a stellar atmosphere with finite gyro-viscosity and Hall currents

S. L. MAHESHWARI AND P. K. BHATIA

Department of Mathematics, University of Jodhpur, Jodhpur

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The convective instability of a thermally unstable stellar atmosphere is considered in the presence of effects of finite Larmor radius and Hall currents, taking into account the possibility of radiative heat transfer. The atmosphere is assumed to be permeated by a uniform vertical magnetic field and to be rotating uniformly about the vertical axis. It is found that the criterion for monotonic instability is not affected by the presence of these effects.

1. INTRODUCTION

Defouw (1970) studied the convective instability (in which motions are driven by buoyancy forces) of a thermally unstable stellar atmosphere. Designating this type of instability as a thermal convective instability, he has shown that a thermally unstable atmosphere is also convectively unstable. He has also shown that the criterion for monotonic instability is given by

$$\frac{1}{C_p}(L_T-\rho\alpha L_\rho)+\chi k^2<0 \qquad \ldots \quad (1)$$

Further he has included the effects of radiative transfer on this instability problem. The source functions S which lead to the thermal-convective instability are found to satisfy the inequality

$$(S_T-\rho\alpha S_\rho)<0 \qquad \ldots \quad (2)$$

The symbols in eqs. (1) and (2) are defined as follows : C_p is specific heat at constant pressure, ρ is density, k is wave number, T is temperature, L is the heat-loss function, χ is the coefficient of thermometric conductivity and α is the coefficient of thermal expansion. The subscripts on L and S denote partial derivatives of these functions with respect to T and ρ evaluated in the equilibrium state.

The effects of a uniform rotation and a uniform magnetic field were examined separately by him and simultaneously Bhatia (1971a).

Bhatia (1971b) also included the effects of radiative transfer on this problem in the presence of the effects of a uniform rotation and a uniform magnetic field.

The same problem has also been studied more recently by Sharma & Kriti Prakash (1975), Sharma (1974) has also extended the problem. studied by Bhatia (1971a). to include the effects of finite Larmor radius and Hall currents which are conveniently taken into account, respectively, in the plasma stress tensor and generalized Ohm's law. For situations of astrophysical interest these influences were found to have no effect on the basic conditions for occurrence of the convective instability. It may, therefore, be of interest to examine the influence of the effects of F. L. R. and Hall currents on the thermal-convective instability in stellar atmosphere when the effects of radiative transfer are included. This aspect forms the subject matter of the present study. The objective is to determine if the conditions for occurence of the instability are changed by these cooperative effects. The configuration is modelled as an infinite horizontal layer which is rotating uniformly about a vertical axis i.e., $\mathbf{\Omega}$ (0, 0, Ω) and is permeated by a uniform vertical magnetic field $\boldsymbol{H}(0, 0, H)$ and gravity force $\boldsymbol{g}(0, 0, -g)$. It is assumed that the fluid is bounded by two free surfaces, where the medium adjoining the fluid is electrically non-conducting. The radiative transfer effects are included with the help of Eddington approximation A dispersion relationship is derived where shows that the condition for instability remains unchanged by the combined effects of rotation, magnetic field, finite Larmor radius and Hall currents.

2. Perturbation Equations

Let $\delta\rho$, $\delta\boldsymbol{P}$, $\boldsymbol{u}(u, v, w)$ and $\boldsymbol{h}(h_x, h_y, h_z)$ denote the perturbations, respectively, in density ρ, stress tensor $\boldsymbol{P}$, velocity and magnetic field $\boldsymbol{H}$. Let $\boldsymbol{g}$, ν, η, N and e denote the gravitational acceleration, the kinematic viscosity, the resistivity, the electron number density and charge of an electron, respectively. Then the linearized hydromagnetic perturbation equations relevant to the present problem are :

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla\delta\boldsymbol{P}+\rho\nu\nabla^2\boldsymbol{u}+2\rho(\boldsymbol{u}\times\mathbf{\Omega})+\frac{1}{4\pi}(\nabla\times\boldsymbol{h})\times\boldsymbol{H}+\boldsymbol{g}\delta\rho, \quad \ldots \quad (3)$$

$$\nabla\cdot\boldsymbol{u} = 0, \qquad \nabla\cdot\boldsymbol{h} = 0, \quad \ldots \quad (4)$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = (\boldsymbol{H}\cdot\nabla)\boldsymbol{u}+\eta\nabla^2\boldsymbol{h}-\left(\frac{1}{4\pi Ne}\right)\nabla\times[(\nabla\times\boldsymbol{h})\times\boldsymbol{H}], \quad \ldots \quad (5)$$

As in Defouw (1970) and Bhatia (1971b) we shall include the effects of radiative transfer with the help of Eddington approximation, the usefulness of which in gas dynamics has been pointed out by Unno & Spiegel (1966). These authors have also emphasized the accuracy of the approximation in both the optically thick and optically thin limits. By an elementary generalization of a formula given by these authors one can derive the following heat equation for a gray gas :

$$\rho C_v \frac{dT}{dt} - \frac{p}{\rho}\frac{d\rho}{dt} = \nabla\cdot\left\{\frac{1}{3\varkappa\rho}\nabla\left[\frac{1}{\varkappa\rho}\left(\rho C_v\frac{dT}{dt} - \frac{p}{\rho}\frac{d\rho}{dt}\right) + 4\pi S\right]\right\}, \qquad \ldots \quad (6)$$

where p is pressure, C_v is specific heat at constant volume, $\varkappa$ is mass absorption coefficient and d/dt is the mobile operator. Supposing that the perturbation in the temperature of a uniform medium is denoted by θ and using the Boussinesq equation of state

$$\frac{\delta\rho}{\rho} = -\alpha\theta, \qquad \ldots \quad (7)$$

and following Defouw (1970), we obtain the linearized perturbation form of (6) as

$$3\varkappa^2\rho^2\frac{\partial\theta}{\partial t} + \frac{4\pi\varkappa}{C_p}(S_T - \rho\alpha S_\rho)\nabla^2\theta - \frac{\partial}{\partial t}\nabla^2\theta +$$

$$+\left(\beta + \frac{g}{C_p}\right)(3\varkappa^2\rho^2 w - \nabla^2 w) = 0, \qquad \ldots \quad (8)$$

In eq. (8) $\beta = (dT/dz)$ is the temperature gradient and ∇^2 is the three-dimensional Laplacian operator. Here we use the cartesian coordinates with z-axis in the vertically upward direction.

Taking into account the finiteness of the ion Larmor radius, the compnents of the stress tensor $\boldsymbol{P}$ are given by (Roberts & Taylor, 1962)

$$\left.\begin{aligned}
\delta P_{xx} &= \delta p - \rho\nu_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),\\
\delta P_{xy} &= \delta P_{yx} = \rho\nu_0\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right),\\
\delta P_{xz} &= \delta P_{zx} = -2\rho\nu_0\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),\\
\delta P_{yy} &= \delta p + \rho\nu_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),\\
\delta P_{yz} &= \delta P_{zy} = 2\rho\nu_0\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right),\\
\delta P_{zz} &= \delta p
\end{aligned}\right\} \qquad \ldots \quad (9)$$

Here δp is the scalar part of the perturbation in pressure p and $\rho\nu_0 = N'T/4\omega_{H'}$, ω_H is the ion-gyration frequency, while N' and T' are, respectively, the number density and temperaure of ions.

Using eqs (4) and (9) and taking once the curl of eq. (5) and twice the curl of eq. (3), we finally obtain the following linearized perturbztion equations :

$$\frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \eta\nabla^2 h_z - \left(\frac{H}{4\pi Ne}\right)\frac{\partial \xi}{\partial z}, \qquad \ldots (10)$$

$$\frac{\partial \xi}{\partial t} = H\frac{\partial \zeta}{\partial z} + \eta\nabla^2\xi + \left(\frac{H}{4\pi Ne}\right)\nabla^2\frac{\partial h_z}{\partial z}, \qquad \ldots (11)$$

$$\frac{\partial \zeta}{\partial t} = \nu\nabla^2\zeta + \frac{H}{4\pi\rho}\frac{\partial \xi}{\partial z} + 2\Omega\frac{\partial w}{\partial z} - \nu_0\left(\nabla^2 - 3\frac{\partial^2}{\partial z^2}\right)\frac{\partial w}{\partial z}, \qquad \ldots (12)$$

$$\frac{\partial}{\partial t}\nabla^2 w = g\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + \nu\nabla^4 w + \frac{H}{4\pi\rho}\frac{\partial}{\partial z}\nabla^2 h_z$$

$$-2\Omega\frac{\partial \zeta}{\partial z} + \nu_0\left(\nabla^2 - 3\frac{\partial^2}{\partial z^2}\right)\frac{\partial \zeta}{\partial z}, \qquad \ldots (13)$$

In these equations ζ and $\xi/4\pi$ are, respectively, the vertical components of vorticity and the current density induced by the perturbation in $\boldsymbol{H}$.

Eq (8) and (10)-(13) have to be solved subject to the appropriate boundary conditions. Here we consider the case in which both the boundaries are free and the medium adjoining the fluid is non-conducting. The case of free boundaries is simplest, mathematically, as also the most appropriate for stellar atmospheres (Spiegel 1965). The boundary conditions that must be satisfied by w, θ, ζ and ξ are (Chandrasekhar, 1961)

$$w = 0, \quad \theta = 0$$

$$\frac{\partial \zeta}{\partial z} = 0, \quad \frac{\partial^2 w}{\partial z^2} = 0 \qquad \ldots (14)$$

$\xi = 0$ and $\boldsymbol{h}$ is continuous with an external vacuum field.

3. Dispersion Relation

We seek solutions of the eqs. (10)-(13) by analyzing in terms of normal modes whose dependence on space coordinates (x, y, z) and time t is of the form

$$\exp\{i(nt).\exp(k_x x + k_y y)\}\sin k_z z, \qquad \ldots (15)$$

where k_z is an integral multiple of π divided by the thickness of the fluid layer, $k = (k_x^2+k_y^2+k_z^2)^{\frac{1}{2}}$ and n is the frequency of the perturbation.

Eliminating ζ, ξ, h_z and θ from eqs (8) and (10)-(13) we obtain an equation in w which, when expression (15) is used, yields the dispersion relation

$$\Big[\Big\{(n+\eta k^2)^2+\left(\frac{H}{4\pi Ne}\right)^2 k^2k_z^2\Big\}\times\Big\{\Gamma\left(\beta+\frac{g}{C_p}\right)+(n+\nu k^2)(RS_T-\rho\alpha S_p+n)\Big\}+$$

$$+k_z^2V^2(n+\pi k^2)(n+RS_T-\rho\alpha S_p)\Big\}\times$$

$$\Big[(n+\nu k^2)\Big\{(n+\eta k^2)^2+k_z^2k^2\left(\frac{H}{4\pi Ne}\right)^2\Big\}+(n+\eta k^2)k_z^2V^2\Big]+$$

$$+\frac{k_z^2}{k^2}(n+RS_T-\rho_\alpha S_p).\quad\Big[(2\Omega+\nu_0 k^2-3k_z^2)\times$$

$$\Big\{(n+\eta k^2)^2+\left(\frac{H}{4\pi Ne}\right)^2k^2k_z^2\Big\}+k^2k_z^2V^2\left(\frac{H}{4\pi Ne}\right)\Big]^2=0. \qquad \dots \quad (16)$$

where $\Gamma=\dfrac{g\alpha(k_x^2+k_y^2)}{k^2}$, $R=\dfrac{4\pi\varkappa}{C_p}\left(1+\dfrac{3\varkappa^2\rho^2}{k^2}\right)^{-1}$, $V^2=\dfrac{H^2}{4\pi\rho}$.

The effects of viscosity and resistivity are negligible in many cases of astrophysical interest. Setting $\eta=\nu=0$, the dispersion relation (16) reduces to

$$n^7+R(S_T-\rho\alpha S_p)n^6+A_5n^5+A_4n^4+A_3n^3+A_2n^2+A_1n+A_0=0. \qquad \dots \quad (17)$$

where

$$A_5=\Gamma\left(\beta+\frac{g}{C_p}\right)+k_z^2\Big\{2V^2+2k^2\left(\frac{H}{4\pi Ne}\right)^2+\frac{R_f^2}{k^2}\Big\}, \qquad \dots \quad (18)$$

$$A_4=Rk_z^2(S_T-\rho\alpha S_p)\Big\{2V^2+2k^2\left(\frac{H}{4\pi Ne}\right)^2+\frac{R_f^2}{k^2}\Big\}, \qquad \dots \quad (19)$$

$$A_3=k_z^2\Big[k_z^2\Big\{V^2+\left(\frac{H}{4\pi Ne}\right)^2\Big\}^2+\Gamma\left(\beta+\frac{g}{C_p}\right)\Big\{V^2+2k^2\left(\frac{H}{4\pi Ne}\right)^2\Big\}+$$

$$+2k_z^2R_f\left(\frac{H}{4\pi Ne}\right)\Big\{V^2+R_f\left(\frac{H}{4\pi Ne}\right)\Big\}\Big]. \qquad \dots \quad (20)$$

$$A_2=k_z^4R(S_T-\rho\alpha S_p)\Big[\Big\{V^2+\left(\frac{H}{4\pi Ne}\right)^2\Big\}^2+$$

$$+2R_f\left(\frac{H}{4\pi Ne}\right)\Big\{V^2+\left(\frac{H}{4\pi Ne}\right)R_f\Big\}\Big], \qquad \dots \quad (21)$$

$$-k_z^4 k^2 \left(\frac{H}{4\pi Ne}\right)^2 \left[\Gamma\left(\beta+\frac{g}{C_p}\right)\left\{V^2+k^2\left(\frac{H}{4\pi Ne}\right)^2\right\}\right.$$

$$\left. k_z^2\left\{V^2+R_j\left(\frac{H}{4\pi Ne}\right)\right\}^2\right]. \quad \ldots \quad (22)$$

$$A_0 = k_z^6 k^2 \left(\frac{H}{4\pi Ne}\right)^2 \left\{V^2+R_f\left(\frac{H}{4\pi Ne}\right)\right\} R(S_T-p\alpha S_p). \quad (23)$$

$$R_f = [2\Omega+\nu_0(k^2-3k_z^2)]. \quad \ldots \quad (24)$$

Equation (17) has a positive real root, leading to monotonic instability, if $(S_T - p\alpha S_p) < 0$ which is precisely the condition (2).

We thus conclude that the condition for instability remains unaffected by the presence of the effects of rotation, magnetic field, F.L.R. and Hall currents.

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